

# Understanding cuprate superconductors with spontaneous nodal gap generation

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We study the spontaneous gap generation for gapless nodal fermions within an effective gauge field theory of high temperature superconductors. When superconductivity appears, the gauge boson acquires a finite mass via Anderson-Higgs mechanism. Spontaneous nodal gap generation takes place if the gauge boson mass  $\xi$  is zero or less than a critical value  $\xi_c$  but is suppressed by a larger gauge boson mass. The generated nodal gap prohibits the appearance of low-energy fermion excitations and leads to antiferromagnetic order. Using the fact that gauge boson mass  $\xi$  is proportional to superfluid density and doping concentration, we build one mechanism that provides a unified understanding of the finite single particle gap along the nodal direction in lightly doped cuprates, the competition and coexistence of antiferromagnetism and superconductivity, and the thermal metal-to-insulator transition from the superconducting state to the field-induced normal state in underdoped cuprates.

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## I. INTRODUCTION

Recently, angle-resolved photoemission spectroscopy (ARPES) measurements were performed on lightly doped cuprates and found a full gap over the whole Brillouin zone<sup>1</sup>. This finding is unexpected since the  $d$ -wave symmetry of the gap/pseudogap of high temperature superconductors has widely accepted<sup>2</sup> and previous extensive ARPES measurements always revealed gapless excitations in the  $(\pm\pi/2, \pm\pi/2)$  direction<sup>3</sup>. The nodal gap decreases upon doping and disappears when superconductivity emerges as the ground state. In some materials with superconductivity being its ground state, the nodal gap is also observed. This indicates that there seems to be a competition and possible coexistence between the nodal gap and superconductivity. This behavior is quite similar to the evolution of orders with doping concentration. Antiferromagnetism is the ground state of undoped and lightly doped cuprate superconductors, but disappears when superconductivity emerges, thus there is a competition between antiferromagnetism and superconductivity. Because of this competition, when superconductivity is suppressed, say by magnetic fields, antiferromagnetism has a chance to appear in a superconductor. The field-induced local antiferromagnetic order has been confirmed by recent neutron scattering and scanning tunnel microscopic (STM) experiments<sup>4,5,6,7</sup>. In particular, antiferromagnetic order is found not only in the Abrikosov vortices but also in the superconducting region around the vortices. This finding strongly supports the coexistence of antiferromagnetism and superconductivity. On the other hand, extensive heat transport measurements show that a residual linear term exists in the whole superconducting region at  $T \rightarrow 0$ , but it decreases with decreasing doping concentration and approaches zero as superconductivity disappears<sup>8,9,10,11,12</sup>. When an external magnetic field is present perpendicular

to the  $\text{CuO}_2$  plane, the thermal conductivity of underdoped cuprates decreases with increasing magnetic field and finally vanishes when the magnetic field is beyond the up critical field  $H_{c2}$ . Thus cuprate superconductors exhibit a thermal metal-to-insulator transition upon going from the superconducting state to the field-induced normal state<sup>10</sup>.

We believe that the above phenomena are universal to all high temperature superconductors and more importantly they are governed by the *same* physical mechanism. In this paper, we argue that all these experimental results can be understood if the gapless nodal fermions acquire a finite gap. Such a spontaneous gap generation for nodal fermions is achieved by coupling the fermions to a gauge field which naturally appears as a result of strong electron correlation effect when we go beyond the slave-boson mean-field treatment of  $t$ - $J$  model.

Spin-charge separation and emergent gauge fluctuation are two key concepts in our scenario. When spin and charge degrees of freedom are separated, the excitations are spin-carrying spinons and charge-carrying holons rather than ordinary electrons. The pairing of spinons is responsible for the observed  $d$ -wave energy gap/pseudogap. The  $d$ -wave gap vanishes at the nodes, so the low-energy fermion excitations are effectively gapless and hence can be described by relativistic massless Dirac fermions. Superconductivity is realized once the holons undergo Bose condensation at low temperatures. The spinons are connected to the holons due to the exchange of an emergent gauge field although they do not interact directly. Thus, the low-energy behavior is dominated by an interacting system consisting of gapless Dirac fermions, holons and an emergent gauge field.

A finite nodal gap is generated spontaneously when the gauge field binds gapless nodal fermions into stable fermion-anti-fermion pairs. Exciting single fermions from the nodal direction of the Brillouin zone requires a finite

energy cost which is responsible for the finite nodal gap observed in ARPES experiments. Since the single particle spectrum is fully gapped in the whole momentum region, no free fermions can exist at low temperatures and consequently there should not be a linear term for the thermal conductivity, which is present in a pure  $d$ -wave superconductor with gap nodes. Spontaneous nodal gap generation can take place if the holons are absent or free. When superconductivity emerges as the ground state, the local gauge symmetry is broken by the holon condensation and the gauge boson becomes massive via Anderson-Higgs mechanism. We found a critical value for the gauge boson mass. Spontaneous nodal gap generation takes place if the gauge boson mass is less than the critical value but is suppressed when the gauge boson mass becomes larger than the critical value. Then there is a competition between spontaneous nodal gap generation and superconductivity. The linear term for thermal conductivity in the superconducting state observed by heat transport measurements reflects the suppression of spontaneous nodal gap generation by superconductivity. Another important consequence of spontaneous nodal gap generation is that it enhances the antiferromagnetic spin correlation greatly and actually leads to long-range order. Thus the antiferromagnetic order should have the same doping dependence with the nodal fermion gap: it is present in low doping region and disappears in high doping region after superconductivity emerges. This is exactly what happens in cuprates superconductors. Since the gauge boson mass is proportional to the superfluid density, the coexistence of spontaneous nodal gap generation and a small gauge boson mass leads to an very important result that the antiferromagnetic order can coexist with superconductivity when the superfluid density is less than the critical value. Thus the spontaneous nodal gap generation provides a unified explanation of the experimental results mentioned at the beginning of this paper.

This paper is organized as follows: In Sec. (II), we first give a brief review on spin-charge separation, slavo-boson treatment of  $t$ - $J$  model and the gauge theory approach to cuprate superconductors, then we discuss chiral symmetry breaking in the presence of massive gauge boson and estimate the critical gauge boson mass which separates chiral symmetric and symmetry broken phases. In Sec. (III), we discuss the explanation of the ARPES experiments, the heat transport behavior and the relationship between antiferromagnetism and superconductivity respectively. The paper ends with a summary.

## II. DYNAMICAL GAP GENERATION FOR GAPLESS NODAL FERMIONS

Shortly after the discovery of high temperature superconductors, it was correctly recognized that these materials are doped Mott insulators and hence can not be properly understood without considering the strong cor-

relation effect. Anderson<sup>13</sup> proposed that due to quantum fluctuations the ground state of undoped cuprates is more probably some kind of quantum liquid of spin singlets, called resonating valence bond (RVB) state. The investigation of RVB state was carried out within the  $t$ - $J$  model which is derived from the more general three-band Hubbard model<sup>14</sup>. The strong correlation nature of cuprate superconductors is reflected in a no-double occupancy constraint which says that there is no more than one electron at one lattice site due to the strong Coulomb repulsion between electrons. By decomposing the electron operator  $c_{i\sigma}^\dagger$  to the product of a spinon operator  $f_{i\sigma}^\dagger$  (neutral fermion) and a holon operator  $b_i$  (charged boson)

$$c_{i\sigma}^\dagger = f_{i\sigma}^\dagger b_i, \quad (1)$$

the no-double occupancy constraint can be written as

$$\sum_{\sigma} f_{i\sigma}^\dagger f_{i\sigma} + b_i^\dagger b_i = 1 \quad (2)$$

which is easier to be treated analytically. This decomposition is the crucial step underlying the so-called slave-boson mean field treatment of  $t$ - $J$  model. A four-fermion interaction term appears in the  $t$ - $J$  model after replacing electron operators with spinon operators and holon operators. This term can be treated by introducing three order parameters  $\chi_{ij} = \langle f_{i\sigma}^\dagger f_{j\sigma} \rangle$ ,  $\Delta_{ij} = \langle f_{i\uparrow} f_{j\downarrow} - f_{i\downarrow} f_{j\uparrow} \rangle$ , and  $\eta_{ij} = b_i^\dagger b_j$ . A phase diagram can be obtained based on this mean field approach<sup>15,16</sup>. The flux phase was found to be locally stable<sup>17</sup> and very interesting for its applicability to cuprate superconductors. Due to its  $d$ -wave spinon gap symmetry, the low-energy excitations are actually gapless Dirac fermions<sup>17,18</sup>. The quantum fluctuations around this mean field state includes a massless U(1) gauge field. The low-energy effective behavior of cuprate superconductors thus can be well described by the three-dimensional quantum electrodynamics (QED<sub>3</sub>).

Although the U(1) formulation captures some important properties of high temperature superconductors, it was shown by Wen and Lee<sup>19</sup> not to connect smoothly to the half-filling material in which an exact SU(2) local symmetry was found. An SU(2) treatment of the 2D  $t$ - $J$  model was constructed to describe the physics of undoped and underdoped cuprates in a unified way<sup>19</sup>. This treatment gives rise to several mean-field phase diagrams among which the staggered flux phase is applicable to cuprate superconductors. Taking quantum fluctuations into account leads to a low-energy effective theory<sup>20</sup> that consists of a massless U(1) gauge field, a two-component bosons, and a massless fermions excited from the gap nodes of  $d$ -wave spinon pairs.

### A. Dynamical chiral symmetry breaking

If the fermions are massless the theory respects a chiral symmetry. However, the QED<sub>3</sub> theory has a rather peculiar property that the massless fermions can acquire a dynamically generated mass when its flavor is less than a critical value. The mass term of fermions spontaneously breaks the chiral symmetry which leads to a massless Goldstone boson according to the Goldstone theorem.

At half-filling, the low energy physics is dominated by the interaction of nodal fermions with U(1) gauge fields<sup>20</sup>

$$\mathcal{L}_F = \sum_{\sigma=1}^N \bar{\psi}_{\sigma} v_{\sigma,\mu} (\partial_{\mu} - ia_{\mu}) \gamma_{\mu} \psi_{\sigma}. \quad (3)$$

The Fermi field  $\psi_{\sigma}$  is a  $4 \times 1$  spinor representing the gapless nodal fermions. The  $4 \times 4$   $\gamma_{\mu}$  matrices obey the algebra,  $\{\gamma_{\mu}, \gamma_{\nu}\} = 2\delta_{\mu\nu}$ , and for simplicity, we let  $v_{\sigma,\mu} = 1$  ( $\mu, \nu = 0, 1, 2$ ). At half-filling case, there are no holons or the holons are all confined because of the presence of a very large charge gap.

Chiral symmetry breaking is a nonperturbative phenomenon and can not be obtained within any finite order of the perturbation expansion. The standard approach to this problem is to solve the self-consistent Dyson-Schwinger (DS) equation for the fermion self-energy. The inverse fermion propagator is written as  $S^{-1}(p) = i\gamma \cdot p A(p^2) + \Sigma(p^2)$ ,  $A(p^2)$  is the wave-function renormalization and  $\Sigma(p^2)$  the fermion self-energy. The propagator of a massless fermion is simply  $S^{-1}(p) = i\gamma \cdot p$ .  $A(p^2)$  and  $\Sigma(p^2)$  appear due to the renormalization effect caused by interaction with gauge field. The self-energy function  $\Sigma(p^2)$  represents the interaction induced fermion mass and is determined by a set of DS integral equations. If the DS equation for  $\Sigma(p^2)$  has only vanishing solutions, the fermions remain gapless and the Lagrangian respects the chiral symmetries  $\psi \rightarrow \exp(i\theta\gamma_{3,5})\psi$ , with  $\gamma_3$  and  $\gamma_5$  two  $4 \times 4$  matrices that anticommute with  $\gamma_{\mu}$  ( $\mu = 0, 1, 2$ ). If the DS equation for  $\Sigma(p^2)$  develops a squarely integrable nontrivial solution<sup>21</sup>, then the originally massless fermions acquire a finite mass. The DS equation is extremely complicated and hence can never be treated without making proper approximations. To the lowest order<sup>22</sup>, we take  $A = 1$  neglecting all higher-order corrections and approximate the vertex function by the bare  $\gamma_{\mu}$ . After making these approximations, the DS equation has the form

$$\Sigma(p^2) = \int \frac{d^3k}{(2\pi)^3} \frac{\gamma^{\mu} D_{\mu\nu}(p-k) \Sigma(k^2) \gamma^{\nu}}{k^2 + \Sigma^2(k^2)}. \quad (4)$$

Appelquist *et al.* showed<sup>22,23</sup> that this DS equation has solutions only for  $N < N_c = 32/\pi^2$ . Since the physical fermion flavor is 2 representing the number of spin component, the massless fermions actually acquire a finite mass. This mechanism is called dynamical chiral symmetry breaking and has been studied for many years in particle physics as a possible mechanism to generate fermion

mass without introducing annoying Higgs bosons. The presence of a critical fermion flavor can be understood as follows. The effective coupling of gauge field is proportional to  $1/N$ . It is strong enough to form fermion pairs when the physical fermion flavor is less than  $N_c$ , while for large  $N$  the coupling is too weak.

### B. Effect of gauge boson mass on chiral structure

When holes are doped into the CuO plane, the holons are excited and hence an additional coupling between holons and the gauge field appears. We have shown<sup>24</sup> in a gauge invariant way that the gapless fermions can acquire a finite gap when the additional holons are not Bose condensed. When superconductivity emerges, the gauge boson becomes massive. In order to understand the properties of nodal fermions we need to investigate the effect of the gauge boson mass on chiral symmetry breaking.

We write the action of bosons in the standard Ginsberg-Landau form

$$\mathcal{L}_B = \frac{1}{4m_b} |(\partial_{\mu} - ia_{\mu})b|^2 - \alpha |b|^2 - \frac{\beta}{2} |b|^4. \quad (5)$$

Note that this is not the popular model that has been extensively studied in the literature<sup>20</sup>. In previous treatment, a non-relativistic model has been used to describe the interaction of holons and gauge fields. In such a model, the density fluctuations of holons screen the temporal component of the gauge field which becomes massive and hence is ignored. However, this treatment destroys the gauge invariance of the theory: the result obtained in the Landau gauge is qualitatively different from that in the Feynman gauge<sup>24</sup>. This inconsistency might be a result of using an inappropriate Lagrangian for the holons. At present, it is not possible to derive an effective action for the holons rigorously. In this paper, we use the relativistic scalar QED to describe the holons because it is the most general field theoretic model for a scalar field and it can lead to a gauge invariant critical fermion flavor.

In the Lagrangian,  $\beta$  is always positive while  $\alpha$  can be positive or negative corresponding to normal and superconducting states, respectively. For  $\alpha > 0$ , the holons are free and the Lagrangian  $\mathcal{L}_B$  is invariant under the local U(1) gauge transformation

$$a_{\mu} \longrightarrow a_{\mu} - \partial_{\mu}\theta \quad (6)$$

$$b \longrightarrow e^{i\theta}b, \quad (7)$$

where  $\theta(x)$  is an arbitrary function. When the holons undergo Bose condensation,  $\alpha$  becomes negative and the ground state occurs at

$$\langle b \rangle = b_0 = \sqrt{-\frac{\alpha}{\beta}}. \quad (8)$$

Thus, the local gauge symmetry is spontaneous broken due to ground state degeneracy. We could write the holon field as

$$b(x) = b_0 e^{i\theta(x)}, \quad (9)$$

where  $\theta$  is the phase of the order parameter which is just the gapless Goldstone mode associated with the spontaneous gauge symmetry breaking. The appearance of this gapless mode used to be a big puzzle to physicists since no such modes had been observed in superconductors. This inconsistency could be eliminated by the Anderson-Higgs mechanism<sup>25</sup>. Its essential idea is to perform the following gauge transformation

$$a_\mu \longrightarrow a_\mu + \partial_\mu \theta, \quad (10)$$

taking advantage of gauge freedom of the theory. Then the Goldstone mode  $\theta$  is removed and there appears a term as follows

$$\frac{1}{m_b} b_0^2 a_\mu a^\mu. \quad (11)$$

It is easy to see that the gauge boson now acquires a finite mass

$$\xi = \frac{1}{m_b} \rho_s \quad (12)$$

after absorbing the gapless Goldstone mode. This is the famous Anderson-Higgs mechanism which when generalized to non-Abelian gauge theories constitutes the foundation of the electro-weak Standard Model. The physical meaning of the finite gauge boson mass can be seen from the equation for magnetic field

$$\nabla^2 \mathbf{B} = \xi \mathbf{B}. \quad (13)$$

Comparing this with the standard London equation, we know that  $\xi = \lambda_L^{-2}$  with  $\lambda_L$  the London penetration depth. The solution of this equation is an exponentially damping function indicating that the magnetic field can penetrate the superconductors only to a finite depth equal to the inverse gauge boson mass.

After the gauge boson acquires a finite mass, its coupling strength is weakened and might not be able to form fermion pairs. To testify the correctness of this naive expectation, we now investigate the effect of a finite gauge boson mass on chiral symmetry breaking by studying the DS equation with massive gauge boson propagator and seek the critical coupling constant. The gauge boson propagator in Landau gauge is

$$D_{\mu\nu}(q) = D_T(q^2) \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right), \quad (14)$$

with

$$D_T^{-1}(q^2) = q^2 [1 + \pi(q^2)] + \xi^2. \quad (15)$$

Let  $q$  be the gauge boson momentum, then  $q^2 = (p - k)^2 = p^2 + k^2 - 2pk \cos \theta$ .  $\pi(q^2)$  is the vacuum polarization of the gauge boson, which is originally introduced to overcome the infrared divergence. In the present case, the gauge field has no kinetic energy term and its dynamics comes from integrating out matter fields. As a result, only  $\pi(q^2)$  appear in  $D_T(q^2)$ , so

$$D_T^{-1}(q^2) = q^2 \pi(q^2) + \xi^2. \quad (16)$$

The vacuum polarization consists of two parts corresponding to fermion contribution  $\pi_F$  and holon contribution  $\pi_B$  respectively, which are

$$\pi_F(q^2) = \frac{N}{8|q|}, \quad (17)$$

$$\pi_B(q^2) = \frac{1}{8|q|} \quad (18)$$

to the one-loop approximation (Note that the holon mass is ignored in the polarization  $\pi_B$  since it does not affect the chiral structure<sup>24</sup>). Adding them up leads to the total vacuum polarization

$$\pi(q^2) = \frac{N+1}{8|q|}. \quad (19)$$

Then we have

$$D_T^{-1}(q^2) = q^2 \pi(q^2) + \xi^2 = \frac{N+1}{8}(q + \eta), \quad (20)$$

with

$$\eta = \frac{8\xi^2}{N+1}. \quad (21)$$

Now we can write down the propagator of gauge boson explicitly

$$D_{\mu\nu}(q) = \frac{8}{(N+1)(|q| + \eta)} \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right). \quad (22)$$

We now substitute this expression into Eq. (4). After performing angular integration and introducing an ultra-violet cutoff  $\Lambda$  we have

$$\begin{aligned} \Sigma(p^2) &= \lambda \int_0^\Lambda dk \frac{k \Sigma(k^2)}{k^2 + \Sigma^2(k^2)} \\ &\times \frac{1}{p} \left( p + k - |p - k| - \eta \ln \left( \frac{p + k + \eta}{|p - k| + \eta} \right) \right) \end{aligned} \quad (23)$$

where  $\lambda = 4/(N+1)\pi^2$  serves as an effective coupling constant<sup>26</sup>.

This integral equation can be investigated by bifurcation theory and parameter imbedding method. In order to obtain the bifurcation points we need only to find the eigenvalues of the associated Fréchet derivative of the nonlinear DS equation<sup>26</sup>. Those eigenvalues that have

odd multiplicity are the bifurcation points. The first bifurcation point is just the critical coupling strength at which a nontrivial solution of the DS equation develops<sup>26</sup>. Once we obtained the critical coupling constant, then we can get the critical fermion flavor that separates the chiral symmetry breaking phase and the chiral symmetric phase.

Numerical calculations<sup>26</sup> found that the critical fermion number is a monotonically increasing function of  $\Lambda/\eta$ . It conforms the naive expectation that a finite mass of the gauge boson is repulsive to gap generation for fermions. For small  $\Lambda/\eta$  the critical number is smaller than physical fermion number 2, so fermions remain gapless. When  $\Lambda/\eta$  increases, the critical number increases accordingly and finally approaches a constant value larger than 2. Thus we can conclude that the spontaneous gap generation for fermions takes place when the gauge boson mass is zero and very small but is destroyed when the gauge boson mass is larger than a critical value. Including the wave function renormalization  $A(p^2)$  does not change this behavior, but changes the critical value quantitatively. We have performed calculations in an appropriate non-local gauge after taking  $A(p^2)$  into account and found that  $\frac{\eta_c}{\Lambda}$  is between  $10^{-4}eV$  and  $10^{-3}eV$ . Although it is hard to determine its exact value, we will show that this uncertainty only has minor influence on the critical values of observable quantities such as the superfluid density and the doping concentration.

We know from these results that there is a competition between two kinds of spontaneous symmetry breaking: chiral symmetry breaking and spontaneous gauge symmetry breaking. Chiral symmetry breaking is associated with the mass generation of massless fermions, while spontaneous gauge symmetry breaking is caused by holon condensation and generates a mass for the gauge boson. If the gauge boson mass is less than the critical value but nonzero, there is a coexistence of chiral symmetry breaking and spontaneous gauge symmetry breaking.

### C. Critical point between chiral symmetric and symmetry breaking phases

We now would like to discuss the critical value for the gauge boson mass since it plays a crucial role in determining the transition between chiral symmetric and symmetry breaking phases. The gauge boson mass is not a physical quantity that can be observed by experiments. In order to explain experiments with our mechanism, we should make a connection between the gauge boson mass with the superfluid density and the doping concentration, which are important physical quantities in describing the superconductors.

The relationship between gauge boson mass with superfluid density we obtained in Eq. (12) is very useful in describing the effect of superconducting condensation on spontaneous gap generation for the nodal quasiparticles. Now we wish to relate the gauge boson mass to dop-

ing concentration  $\delta$ . Careful London penetration depth measurements and optical conductivity experiments<sup>27</sup> showed that the superfluid density is proportional to the doping concentration, that is

$$\rho_s(T=0) = \frac{\delta}{a^2}, \quad (24)$$

where  $a$  is the lattice constant. The optical conductivity experiments reflects the response of the system to electromagnetic field, instead of the internal gauge field. This seems to be an obstacle to use the above equation. However, we expect it works well for both the internal gauge field and the electromagnetic field since they have the same gauge structure. Thus, we obtain the relationship between gauge boson mass and doping concentration

$$\delta = m_b a^2 \xi. \quad (25)$$

Based on this formula, the doping dependence of many physical quantities can be described by their dependence of the gauge boson mass. Note that this property is special to cuprate superconductors which are believed to be doped Mott insulators.

Next we would like to calculate the critical value of the doping concentration which separates the chiral symmetric and symmetry breaking phases. As will be seen in the following discussions, this is an important quantity in understanding experiments. Our calculation of DS equation has given the critical gauge boson mass. From the numerical results in the nonlocal gauge<sup>26</sup>, we know that  $\frac{\eta_c}{\Lambda}$  is between  $10^{-4}eV$  and  $10^{-3}eV$ . The continuum U(1) gauge theory is the effective low-energy theory of high temperature superconductors, hence the lattice provides a natural ultraviolet cutoff. However, we can also choose  $\alpha = (N+1)/8$  as the ultraviolet cutoff since in QED<sub>3</sub> all physical quantities damps rapidly above this energy scale<sup>22</sup>. Remember that we have defined  $\eta = 8\xi^2/(N+1)$ . Putting all these together, we get the critical doping concentration

$$\delta_c = \frac{\xi_c}{2(2m_b a^2)^{-1}}. \quad (26)$$

The mass of the holons is determined by the hopping integral  $t$  in the  $t$ - $J$  model. In the tight-binding treatment, we have

$$(2m_b a^2)^{-1} = t_h = 0.122eV, \quad (27)$$

which was obtained by Lee and Wen<sup>28</sup> in the case of YBCO<sub>6.95</sub>. If  $\frac{\eta_c}{\Lambda} = 10^{-4}eV$  then the critical doping concentration is  $\delta_c = 0.03$ ; while if  $\frac{\eta_c}{\Lambda} = 10^{-3}eV$ , then  $\delta_c = 0.05$ . Since the antiferromagnetic order disappears generally at 0.03, the critical  $\delta_c$  we obtained is in good agreement with experiments and the inability in determining the exact value of  $\frac{\eta_c}{\Lambda}$  does not affect the reliability of our conclusion.

Spontaneous gap generation takes place for doping concentration less than  $\delta_c$ , no matter the ground state is

superconducting or not. For doping concentration larger than  $\delta_c$ , if the ground state is not superconductivity, then spontaneous gap generation also takes place; however, if the ground state is superconducting, then spontaneous nodal gap generation is suppressed by superconductivity. In the superconducting state with doping concentration higher than  $\delta_c$ , there is possibility that the superfluid density is reduced to below its critical value  $\rho_{sc}$  by some means other than decreasing doping. If this really happens, then spontaneous gap generation can also take place. This coexistence of superconductivity and spontaneous gap generation is very important and will be discussed in the next section.

### III. EXPLAINING EXPERIMENTS WITH SPONTANEOUS NODAL GAP GENERATION

Since we know the effect of holons and gauge boson mass on the gap structure of nodal fermions, we are equipped for understanding the exotic experimental findings we have mentioned in the Introduction. These experiments include the finite single particle gap along the nodal direction, the low temperature quasiparticle heat transport, the doping dependence of antiferromagnetism and its relationship with superconductivity. The common feature of these phenomena is that they are all dominated by the behavior of nodal quasiparticles. It is thus not surprised that they can be understood by one single physical mechanism. Although our mechanism is still rather qualitative, its efficiency in explaining these experimental results gives us confidence that it does capture some essential physics of high temperature superconductors.

#### A. Finite energy gap in nodal direction

Now we discuss the application of our theory to ARPES experiments. In RVB theories, all the single particle gaps are caused by the pair formation of spinons. The spinon gap exists at any doping concentration that is less than optimal doping and has  $d$ -wave symmetry. ARPES measurements play an significant role in studying the gap symmetry since it is momentum dependent. Most previous ARPES measurements have been limited to optimally doped  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+x}$  because these materials have ideal surfaces which are required by ARPES measurements. However, to understand high temperature superconductivity, it is necessary to know the electronic structure of underdoped, lightly doped and undoped cuprates.

Recently, ARPES measurements have been performed in the lightly doped cuprates including hole-doped  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  and  $\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$  and electron-doped  $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4$ <sup>1</sup>. The spectra from  $\text{Ca}_{2-x}\text{Na}_x\text{CuO}_2\text{Cl}_2$  reveals that a finite gap exists in the nodal direction at  $x = 0.05$  and becomes smaller with

increasing doping concentration. It closes at  $x = 0.12$  corresponding to a critical temperature  $T_c = 22\text{K}$ . For  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ , a clear nodal gap is observed at doping concentrations  $x = 0.01$  and  $x = 0.02$ , but it closes at  $x = 0.03$  which is less than the critical doping  $x_c = 0.05$  where superconductivity starts to emerge as the ground state. In the case of slightly electron-doped  $\text{Nd}_{2-x}\text{Ce}_x\text{CuO}_4$ , the gap in the nodes is observed at doping  $x = 0.025$  and  $x = 0.04$ , and it does not exist at doping  $x = 0.08$  and  $x = 0.10$ , well below the critical doping concentration  $x_c = 0.12$ . Such a doping dependence of the excitation gap in the nodal direction is observed in three different cuprate superconductors and hence should be a universal phenomena. These observations are rather striking because based on the  $d$ -wave symmetry of gap/pseudogap, the energy gap should vanish along the nodal directions.

Although the nodal gap in the three cuprates has the same dependence of the doping concentration, the inconsistency of critical doping at which the nodal energy gap closes and critical doping at which superconductivity emerges brings a difficulty to understand the relationship between the nodal gap and high temperature superconductivity. However, the difficulty is not that severe as it seems to be, because all the above ARPES measurements are performed at a finite temperature  $T = 15\text{K}$  rather than nearly zero temperature. It is very probable that the true ground state of all the lightly doped cuprates has the same electronic structure and the above ARPES observed inconsistency is caused by thermal fluctuations which are different from material to material.

We speculate that this picture is what really happens in high temperature superconductors. In the spirit of our results, at doping concentration that is lower than the critical doping at which superconductivity appears, the instability caused by strong gauge fluctuations always generates a finite gap for the nodal fermions. When superconductivity emerges, if its superfluid density is less than some critical value, the nodal fermions also have a finite gap. This finite gap is suppressed by a larger doping concentration, i.e., a larger superfluid density. This is a universal picture for the evolution of zero-temperature fermion energy spectrum along the nodal direction in all cuprate superconductors. However, the thermal fluctuations are different in different materials and they destroy the spontaneously generated nodal gap at different temperatures. This is qualitatively in agreement with the ARPES data of Shen's group<sup>1</sup>. To make quantitative comparison with experimental data, detailed calculations considering chemical structure and disorders are needed, which are beyond the scope of this paper.

In order to compare our results with experiments, the above discussions on dynamical fermion gap generation should be extended to finite temperatures. The problem has been investigated by several authors with the results that above a critical temperature the chiral symmetry is restored and the fermions remain gapless<sup>29</sup>. Before the holes are doped into the Cu-O plane, the physical fermion flavor is  $N_f = 2$ . According to the results of Aitchison *et*

*al.*,  $\frac{k_B T_c}{N_f} = 0.002$ , which leads to  $T_c = 45K$ . From Eq. (19) we know that the holons shift the effective flavor to  $N_f = 3$ . From the result of Aitchison *et al.* we know that  $\frac{k_B T_c}{N_f}$  is about  $10^{-4}$ , which leads to  $T_c \sim 4K$ . The nodal gap generation occurs only at temperatures below this critical value. When the temperature is beyond this critical value, thermal fluctuations destroy the nodal gap generation and hence the gap along the nodal direction closes.

### B. Competition and coexistence of antiferromagnetism and superconductivity

Understanding the competition of various ground states of cuprate superconductors is one of the central problems in modern condensed matter physics. The cuprate superconductors in its half-filling limit are believed to be Mott insulators with long-range antiferromagnetic order. When the doping concentration increases, long-range AF order becomes short-ranged and *d*-wave superconductivity emerges as the ground state. It is interesting to build a microscopic theory to describe the evolution from the antiferromagnetism phase to the superconductivity phase with doping.

The doping dependence of antiferromagnetism is very similar to that of the nodal gap observed in ARPES measurements, indicating that they might be governed by the same mechanism. This can be easily verified by calculating the antiferromagnetic spin correlation function. Once the nodal fermion acquires a finite gap, the antiferromagnetic spin correlation is greatly enhanced. Actually, it has been argued that the chiral symmetry breaking corresponds to the formation of antiferromagnetic long-range order<sup>20,24,26,30</sup>. The gapless spin wave excitation is interpreted as the massless Goldstone boson. The antiferromagnetic spin correlation is defined as

$$\langle S^+ S^- \rangle = -\frac{1}{4} \int \frac{d^3 k}{(2\pi)^3} \text{Tr} [G_0(k) G_0(k+p)], \quad (28)$$

where  $G_0(k)$  is the fermion propagator. If the fermions are massless, then  $G_0(k) = \frac{-i}{\gamma \cdot k}$  and we have  $\langle S^+ S^- \rangle = -\frac{|p|}{16}$ . At  $p \rightarrow 0$ ,  $\langle S^+ S^- \rangle_0 \rightarrow 0$ , and the antiferromagnetic correlation is heavily lost. The propagator for the massive fermion is

$$G(k) = \frac{-(\gamma \cdot k + i\Sigma_0)}{k^2 + \Sigma_0^2}, \quad (29)$$

where a constant mass  $\Sigma_0$  is adopted which does not affect the conclusion. This propagator leads to a correlation function  $\langle S^+ S^- \rangle$  that behaves like  $-\Sigma_0/2\pi$  as  $p \rightarrow 0$  and we have long-range antiferromagnetic correlation when chiral symmetry breaking takes place<sup>26</sup>.

The gauge boson mass in the superconducting state is an appropriate physical quantity to describe superconductivity. Thus the relationship between antiferromagnetism and superconductivity can be described by

the relationship between spontaneous nodal gap generation and the mass of gauge boson. We have shown that spontaneous nodal gap generation can take place for doping concentration less than  $\delta_c$ . This indicates that antiferromagnetism only exists at half-filling and lightly doped cuprates, in consistent with experiments. In most cuprate superconductors, the superfluid density  $\rho_s$  is large enough to suppress antiferromagnetism once superconductivity emerges at the critical doping concentration, which is generally larger than  $\delta_c$ . Hence, antiferromagnetism can not coexist with superconductivity in the bulk materials, at least in most cuprate superconductors. However, if an external magnetic field is introduced to the superconductors and reduces the superfluid density  $\rho_s$  down to below its critical value, then antiferromagnetism intends to appear in the superconducting state. The coexistence of antiferromagnetism and superconductivity is thus possible.

Recently, intense investigation have been taken on the magnetic field induced local antiferromagnetic order. The external magnetic field perpendicular to the  $\text{CuO}_2$  plane generates Abrikosov vortices inside which the superfluid density is suppressed. Around the vortex cores, if the superfluid density is reduced to below the critical value, then spontaneous gap generation and hence antiferromagnetic order can be formed in the vortex state<sup>4,5,6,7</sup>. Demler *et al.* have attempted to explain these experiments by assuming a proximity to the coexistence of spin-density wave and superconductivity<sup>31</sup>. Such a coexistence can also be addressed within several other theories<sup>32,33,34,35</sup> including Zhang's  $\text{SO}(5)$  theory<sup>32</sup> and the staggered flux approach proposed by Lee and Wen<sup>33</sup>. In this paper we use spin-charge separation and dynamical chiral symmetry breaking to account for the competition and coexistence of antiferromagnetism and superconductivity. Our approach emphasizes on the similarity of this competition and coexistence with the transport behavior and single particle spectrum properties. More quantitative calculation will be carried out in the future in order to produce the detailed experimental data.

### C. Low temperature heat transport behavior

The low temperature transport behavior of superconductors is controlled by the gap symmetry since it determines the type of low-energy excitations. For conventional *s*-wave superconductors, the low-energy density of states  $N_s(\omega)$  damps rapidly as temperature decreases, i.e.,  $N_s(\omega) = 0$  for  $|\omega| < \Delta_0$  with  $\Delta_0$  the quantity of energy gap. However, the situation is quite different for *d*-wave superconductors due to the presence gap nodes. In the absence of impurities, the density of states is  $N_d(\omega) \sim \omega$ . Due to the scattering of impurities a finite density of quasiparticle states exist, i.e.,  $N_d(0)$  is finite<sup>36</sup>. Lee<sup>37</sup> has investigated the quasiparticle transport of *d*-wave superconductors and found that it is independent of impurity intensity as a result of the competition be-

tween the growth of quasiparticle density of states and the reduction of mean free path. In contrast to  $s$ -wave superconductors where thermal conductivity vanishes due to the absence of mobile fermions at  $T \rightarrow 0$ , a linear term for thermal conductivity appears in  $d$ -wave superconductors at  $T \rightarrow 0$ . Taillefer *et al.* for the first time observed such a linear term for thermal conductivity in optimally doped cuprate  $\text{YBa}_2\text{Cu}_3\text{O}_{6.9}$ <sup>8</sup>. A similar linear term is also observed<sup>9</sup> in optimally doped  $\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8$ . Recently, extensive heat transport measurements have been performed in  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  and  $\text{YBa}_2\text{Cu}_3\text{O}_y$  in a wide range of doping concentration from overdoping to very underdoping. A linear term for thermal conductivity is observed through the whole superconducting region<sup>11,12</sup>. Decreasing the doping concentration drives the cuprate to the proximity to a critical point at which spontaneous gap generation occurs. As a consequence, the thermal conductivity should decrease down to zero at low doping. In the spirit of our mechanism, the gapless nodal quasiparticles in the superconducting state are stable against gauge fluctuations because the gauge boson becomes massive and hence can not generate a finite gap for the gapless nodal fermions. For doping concentration below  $\delta_c$ , the nodal quasiparticles acquire a finite gap which changes the node structure of  $d$ -wave gap symmetry and suppresses the appearance of low-energy fermions. Therefore, there is no linear term for the thermal conductivity at very low doping region.

Recently the dependence of thermal transport on magnetic field has been investigated. Naively, magnetic field should drive the thermal conductivity to increase since it breaks the Cooper pairs and hence quasiparticle density increases with magnetic field. This is true for many superconductors including conventional  $s$ -wave superconductors and overdoped cuprates. However, this is not true for underdoped  $d$ -wave cuprate superconductors, in which low temperature thermal conductivity decreases with magnetic field. The crucial reason for this difference is that spin-charge separation caused by strong electron correlation takes place in underdoped cuprates but not in overdoped cuprates and conventional BCS superconductors. When spin and charge degrees of freedom are separated, the holon condensation is suppressed by strong magnetic fields and consequently the mass of internal gauge boson decreases down to zero with increasing magnetic field. On the contrary, the spinon pairs are stable against the external magnetic fields. However, the nodal quasiparticle is affected by the change of internal gauge boson mass. When superconductivity is destroyed by external magnetic fields at  $H_{c2}$ , the mass gap of internal gauge boson vanishes. As a result, the gapless nodal fermions acquire a finite gap. Below this gap the density of states of quasiparticles is zero, preventing the appearance of low energy fermion excitations. Thus we now see that, while magnetic field generates fermion quasiparticles in many ordinary superconductors, the  $d$ -wave underdoped cuprate superconductors has a rather peculiar property that the magnetic field reduces low-energy

fermions due to the spin-charge separation and spontaneous nodal gap generation.

When superconductivity is completely suppressed by magnetic field, heat is transported only by bosons including spin wave, holons and phonons at low temperatures. These bosons can only contribute a  $T^3$  term to thermal conductivity at low temperature, which can be nearly neglected. Therefore, according to our mechanism there should be a thermal metal-to-insulator transition upon going from the superconducting state to the field-induced normal state of underdoped cuprates. This phenomenon has been observed in underdoped  $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$  recently in heat measurements<sup>10</sup>. In contrast to the thermal insulator property of the field-induced normal state, the holons can move freely giving rise an metal-like charge transport behavior. Thus we expect that the Wiedemann-Franz law, which gives a universal relationship between the thermal conductivity  $\kappa$  and the electrical conductivity  $\sigma$  as  $\frac{\kappa}{\sigma T} = \frac{\pi^2}{3} \left(\frac{k_B}{e}\right)^2$  with  $k_B$  the Boltzmann's constant, should no longer hold in this region. Experiments<sup>38</sup> have provided certain evidence supporting the breakdown of this law in the electron-doped cuprate  $\text{Pr}_{2-x}\text{Ce}_x\text{CuO}_{4-y}$ .

#### IV. SUMMARY

One remarkable property of high temperature superconductors is its  $d$ -wave gap symmetry. Due to the presence of gap nodes, there is an amount of low-energy quasiparticles which play a crucial role in determining the low-temperature behavior of cuprate superconductors. For example, the thermally excited nodal quasiparticles can efficiently destroy the superfluid density<sup>28</sup>. While at low temperatures, the nodal quasiparticles contribute a finite thermal conductivity that is independent of impurity concentrations. The  $d$ -wave gap in the single-particle spectrum exists not only in the superconducting state but also in the normal state of underdoped cuprates. The gapless nodal fermion excitations couples to a strong gauge field and can acquire a dynamically generated gap. When this happens, the gap nodes are removed and all excitations are gapped. The spontaneous nodal gap generation modifies the picture of low-temperature physics to a new one, in which no free fermions can be found and a long-range antiferromagnetic order is formed.

The spontaneous generation of nodal gap depends on the gauge boson. If the gauge boson is massless, then the gap generation can always take place. When superconductivity emerges, the gauge boson becomes massive. There is a critical value for the gauge boson mass, only below which could spontaneous gap generation for nodal fermions take place. The high temperature superconductors have a peculiarity that the gauge boson mass is proportional to the doping concentration. Thus spontaneous nodal gap generation should exist at low doping concentrations while vanishes as doping increases. However, when superconductivity is suppressed by some external



means such as external magnetic field, spontaneous nodal gap generation occurs and correspondingly low temperature nodal quasiparticles disappear. Hence there is a thermal metal-insulator transition from the superconducting ground state to the field-induced normal ground state upon increasing magnetic field. On the other hand, spontaneous nodal gap generation corresponds to the formation of antiferromagnetic long-range order. The fate of antiferromagnetism hence can be described by the relationship of spontaneous nodal gap generation with superfluid density or doping concentration.

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